Chapter 8: Universal Gravitation

Practice Problems

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1. An asteroid revolves around the sun with a mean average orbital radius twice that of Earth's. Predict the period of the asteroid in earth years.

$$\left[\frac{T_a}{T_E}\right]^2 = \left[\frac{r_a}{r_E}\right]^3 \text{ with } r_a = 2r_E.$$

$$\text{Thus, } T_a^2 = \left[\frac{r_a}{r_E}\right]^3 T_E^2$$

$$= \left[\frac{2r_E}{r_E}\right]^3 (1\text{ yr})^2 = 8\text{ yr}^2,\ T_a = 2.8\text{ yr}$$

2. From Table 8–1, you can calculate that, on the average, Mars is 1.52 times as far from the sun as is Earth. Predict the time required for Mars to circle the sun in earth days.

$$\left[\frac{T_M}{T_E}\right] = \left[\frac{r_M}{r_E}\right]^3, \text{ with } r_M = 1.52r_E.$$

$$\text{Thus, } T_M^2 = \left[\frac{r_M}{r_E}\right]^3 T_E^2 = \left[\frac{1.52r_E}{r_E}\right]^3 (365\text{ days})^2$$

$$= 4.679 \times 10^5\text{ days}^2,$$

$$T_M = 684\text{ days}$$

3. The moon has a period of 27.3 days and has a mean distance of 3.90×10^5 km from the center of Earth. Find the period of an artificial satellite that is 6.70×10^3 km from the center of Earth.

$$\left[\frac{T_s}{T_m}\right]^2 = \left[\frac{r_s}{r_m}\right]^3,\ T_s^2 = \left[\frac{r_s}{r_m}\right]^3 T_m^2$$

$$= \left[\frac{6.70 \times 10^3\text{ km}}{3.90 \times 10^5\text{ km}}\right]^3 (27.3\text{ days})^2$$

$$= 3.779 \times 10^{-3}\text{ days}^2,$$

$$T_s = 6.15 \times 10^{-2}\text{ days} = 88.5\text{ min}$$

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4. From the data on the period and radius of revolution of the moon in Practice Problem 3, find the mean distance from Earth's center to an artificial satellite that has a period to 1.00 day.

$$\left[\frac{T_s}{T_m}\right]^2 = \left[\frac{r_s}{r_m}\right]^3, \text{ so } r_s^3 = r_m^3\left[\frac{T_s}{T_m}\right]^2$$

$$= (3.90 \times 10^5\text{ km})^3\left[\frac{1.00}{27.3}\right]^2$$

$$= 7.96 \times 10^{13}\text{ km}^3,$$

$$\text{so } r_s = 4.30 \times 10^4\text{ km}$$

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Assume a near–circular orbit for all calculations.

5. a. Calculate the velocity that a satellite shot from Newton's cannon must have in order to orbit Earth, 150 km above its surface.

$$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.52 \times 10^6}}$$

$$= 7.82 \times 10^3\text{ m/s}$$

b. How long would it take for the satellite to return to the cannon in seconds and minutes?

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$

$$= 2\pi\sqrt{\frac{(6.52 \times 10^6\text{ m})^3}{(6.67\times10^{-11}\text{ Nm}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})}}$$

$$= 5.24 \times 10^3\text{ s} = 87.3\text{ min}$$

6. Use the data in Table 8–1 for Mercury to find

a. the speed of a satellite in orbit 265 km above the surface.

$$v = \sqrt{\frac{GM_m}{r}}, \text{ with } r = r_m + 265\text{ km}$$

$$= 2.43 \times 10^6\text{ m} = 0.265 \times 10^6\text{ m}$$

$$= 2.70 \times 10^6\text{ m}$$

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\text{ Nm}^2/\text{kg}^2)(3.2 \times 10^{23}\text{ kg})}{2.70 \times 10^6\text{ m}}}$$

$$= 2.8 \times 10^3\text{ m/s}$$

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b. the period of the satellite.

$$T = 2\pi\sqrt{\frac{r^3}{GM_m}}$$

$$= 2\pi\sqrt{\frac{(2.70 \times 10^6\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(3.2 \times 10^{23}\ \text{kg})}}$$

$= 6.03 \times 10^3\ \text{s} = 1.0 \times 10^2\ \text{min}$

7. a. Find the velocity with which Mercury moves around the sun.

$v = \sqrt{\frac{GM}{r}}$, where here M is the mass of the sun.

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(1.991 \times 10^{30}\ \text{kg})}{(5.80 \times 10^{10}\ \text{m})}}$$

$= 4.79 \times 10^4$ m/s.

b. Also, find the velocity of Saturn. Now, comment on whether or not it makes sense that Mercury is named after a speedy messenger of the gods, while Saturn is named after the father of Jupiter.

$$v = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(1.991 \times 10^{30}\ \text{kg})}{(1.427 \times 10^{12}\ \text{m})}}$$

$= 9.65 \times 10^3$ m/s, about 1/5 as fast as Mercury.

8. We can consider the sun to be a satellite of our galaxy, the Milky Way. The sun revolves around the center of the galaxy with a radius of 2.2×10^{20} m. The period of one rotation is 2.5×10^8 years.

a. Find the mass of the galaxy.

Using $T = 2\pi\sqrt{\frac{r^3}{GM}}$, with

$T = 2.5 \times 10^8 \text{y} = 7.9 \times 10^{15}$ s

$$M = \frac{4\pi^2 r^3}{GT^2}$$

$$= \frac{4\pi^2(2.2 \times 10^{20}\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(7.9 \times 10^{15}\ \text{s})^2}$$

$= 1.0 \times 10^{41}$ kg

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b. Assuming the average star in the galaxy has the mass of the sun, find the number of stars.

$$\text{number of stars} = \frac{\text{total galaxy mass}}{\text{mass per star}}$$

$$= \frac{1.0 \times 10^{41}\ \text{kg}}{2.0 \times 10^{30}\ \text{kg}}$$

$= 5.0 \times 10^{10}$

c. Find the speed with which the sun moves around the center of the galaxy.

$$v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(1.0 \times 10^{41}\ \text{kg})}{2.2 \times 10^{20}\ \text{m}}}$$

$= 1.7 \times 10^5\ \text{m/s} = 6.1$ km/h

Chapter Review Problems

Use $G = 6.670 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

1. Jupiter is 5.2 times farther than Earth is from the sun. Find Jupiter's orbital period in earth years.

$$\left[\frac{T_J}{T_E}\right]^2 = \left[\frac{r_J}{r_E}\right]^3, \text{ so}$$

$$T_J^2 = \left[\frac{r_J}{r}\right]^3 T_E^2$$

$$= \left[\frac{5.2}{1.0}\right]^3 (1.0\ \text{yr})^2 = 141\ \text{yr}^2$$

So $T_J = 12$ yr.

2. Uranus requires 84 years to circle the sun. Find Uranus's orbital radius as a multiple of Earth's orbital radius.

$$\left[\frac{T_U}{T_E}\right]^2 = \left[\frac{r_U}{r_E}\right]^3, \text{ so}$$

$$r_U^3 = \left[\frac{T_U}{T_E}\right]^2 r_E^3$$

$$= \left[\frac{84\ \text{yr}}{1.0\ \text{yr}}\right]^2 (1.0\ r_E)^3 = 7.06 \times 10^3 r_E^3$$

So $r_U = 19\ r_E$

Chapter Review Problems

3. Venus has a period of revolution of 225 earth days. Find the distance between the sun and Venus as a multiple of Earth's orbital radius.

$$\left[\frac{T_V}{T_E}\right]^2 = \left[\frac{r_V}{r_E}\right]^3, \text{ so } r_V^3 = \left[\frac{T_V}{T_E}\right]^2 r_E^3 = \left[\frac{225}{365}\right]^2 r_E^3 = 0.380\ r_E^3. \text{ So } r_v = 0.724\ r_E.$$

4. If a small planet were located 8.0 times as far from the sun as Earth, how many years would it take the planet to orbit the sun?

$$\left[\frac{T_x}{T_E}\right]^2 = \left[\frac{r_x}{r_E}\right]^3, \text{ so } T_x^2 = \left[\frac{r_x}{r_E}\right]^3 T_E^2 = \left[\frac{8.0}{1.0}\right]^3 (1.0\text{ yr})^2 = 512\text{ yr}^2. \text{ So } T_x = 23\text{ yr.}$$

5. A satellite is placed in orbit with a radius that is half the radius of the moon's orbit. Find its period in units of the period of the moon.

$$\left[\frac{T_S}{T_m}\right]^2 = \left[\frac{r_S}{r_m}\right]^3, \text{ so } T_S^2 = \left[\frac{r_S}{r_m}\right]^3 T_m^2 = \left[\frac{0.50\ r_m}{r_m}\right]^3 T_m^2 = \frac{1}{8.0} T_m^2. \text{ So } T_S = 0.35\ T_m.$$

6. An apparatus like the one Cavendish used to find G has a large lead ball that is 5.9 kg in mass and a small one that is 0.047 kg. Their centers are separated by 0.055 m. Find the force of attraction between them.

$$F = \frac{Gm_1m_2}{d^2} = \frac{(6.670 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.9\text{ kg})(4.7 \times 10^{-2}\text{ kg})}{(5.5 \times 10^{-2}\text{ m})^2} = 6.1 \times 10^{-9}\text{ N}$$

7. Use the data in Table 8–1 to compute the gravitational force the sun exerts on Jupiter.

$$F = \frac{Gm_sm_j}{d^2} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.991 \times 10^{30}\text{ kg})(1.901 \times 10^{27}\text{ kg})}{(7.781 \times 10^{11}\text{ m})^2} = 4.17 \times 10^{23}\text{ N}$$

8. Tom has a mass of 70.0 kg and Sally has a mass of 50.0 kg. Tom and Sally are standing 20.0 m apart on the dance floor. Sally looks up and she sees him. She feels an attraction. If the attraction is gravitation, find its size. Assume that both can be replaced by spherical masses.

$$F = \frac{Gm_Tm_S}{d^2} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(70.0\text{ kg})(50.0\text{ kg})}{(20.0\text{ m})^2} = 5.84 \times 10^{-10}\text{ N}$$

9. Two balls have their centers 2.0 m apart. One has a mass of 8.0 kg. The other has a mass of 6.0 kg. What is the gravitational force between them?

$$F = G\frac{m_1m_2}{r^2} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(8.0\text{ kg})(6.0\text{ kg})}{(2.0\text{ m})^2} = 8.0 \times 10^{-10}\text{ N}$$

10. Two bowling balls each have a mass of 6.8 kg. They are located next to one another with their centers 21.8 cm apart. What gravitational force do they exert on each other?

$$F = G\frac{m_1m_2}{r^2} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(6.8\text{ kg})(6.8\text{ kg})}{(0.218\text{ m})^2} = 6.5 \times 10^{-8}\text{ N}$$

11. Sally has a mass of 50.0 kg and Earth has a mass of 5.98×10^{24} kg. The radius of Earth is 6.371×10^6 m.

a. What is the force of gravitation attraction between Sally and Earth?

$$F = \frac{Gm_Sm_E}{d^2} = \frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(50.0\text{ kg})(5.98 \times 10^{24}\text{ kg})}{(6.371 \times 10^6\text{ m})^2} = 491\text{ N}$$

b. What is Sally's weight?

$W = mg = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$

12. The gravitational force between two electrons 1.00 m apart is 5.42×10^{-71} N. Find the mass of an electron.

$F = \dfrac{Gm_1m_2}{d^2}$, but $m_1 = m_2 = m_e$

So $m_e^2 = \dfrac{Fd^2}{G} = \dfrac{(5.42 \times 10^{-71}\ \text{N})(1.00\ \text{m})^2}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 8.13 \times 10^{-62}\ \text{kg}^2$

So $m_e = 9.01 \times 10^{-31}$ kg

13. Two spherical balls are placed so their centers are 2.6 meters apart. The force between the two balls is 2.75×10^{-12} N. What is the mass of each ball if one ball is twice the mass of the other ball?

$F = \dfrac{Gm_1m_2}{d^2}$, but $m_2 = 2m_1$, so $F = \dfrac{G(m_1)(2m_1)}{d^2}$ and $m_1 = \sqrt{\dfrac{Fd^2}{2G}} = \sqrt{\dfrac{(2.75 \times 10^{-12}\ \text{N})(2.6\ \text{m})^2}{2(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}}$

$m_1 = 0.37$ kg.
$m_2 = 2m_1 = 0.75$ kg

14. Using the fact that a 1.0-kg mass weighs 9.8 N on the surface of Earth and the radius of Earth is roughly 6.4×10^6 m,

a. calculate the mass of Earth.

$F = G\dfrac{m_1m_2}{r^2}$

$m_e = \dfrac{Fr^2}{Gm} = \dfrac{(9.8\ \text{N})(6.4 \times 10^6\ \text{m})^2}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg})(1.0\ \text{kg})} = 6.0 \times 10^{24}$ kg

b. calculate the average density of Earth.

$V = \dfrac{4}{3}\pi r^3 = \dfrac{(4\pi)(6.4 \times 10^6\ \text{m})^3}{3} = 1.1 \times 10^{21}\ \text{m}^3$

$D = \dfrac{M}{V} = \dfrac{6.0 \times 10^{24}\ \text{kg}}{1.1 \times 10^{21}\ \text{m}^3} = 5.5 \times 10^3\ \text{kg/m}^3$

15. The moon is 3.9×10^5 km from Earth's center and 1.5×10^8 km from the sun's center. If the masses of the moon, Earth, and sun are 7.3×10^{22} kg, 6.0×10^{24} kg, and 2.0×10^{30} kg, respectively, find the ratio of the gravitational forces exerted by Earth and the sun on the moon.

$F = G\dfrac{m_1m_2}{d^2}$

Earth on moon: $F_e = \dfrac{G(6.0 \times 10^{24}\ \text{kg})(7.3 \times 10^{22}\ \text{kg})}{(3.9 \times 10^8\ \text{m})^2} = 1.9 \times 10^{20}$ N

Sun on moon: $F_s = \dfrac{G(2.0 \times 10^{30}\ \text{kg})(7.3 \times 10^{22}\ \text{kg})}{(1.5 \times 10^{11}\ \text{m})^2} = 4.3 \times 10^{20}$ N

Ratio is $\dfrac{F_e}{F_s} = \dfrac{1.9 \times 10^{20}\ \text{N}}{4.3 \times 10^{20}\ \text{N}} = \dfrac{1.0}{2.3}$.

The sun pulls more than twice as hard on the moon as the Earth.

16. A force of 40.0 N is required to pull a 10.0-kg wooden block at a constant velocity across a smooth glass surface on Earth. What force would be required to pull the same wooden block across the same glass surface on Jupiter? (Jupiter's mass is 1.90 ×10^{27} kg and its radius is 7.18 × 10^7 m.)

$\mu = \frac{F_f}{F_N} = \frac{F_f}{m_b g}$ where m_b is the mass of the block.

On Jupiter the normal force is equal to the gravitational attraction between the block and Jupiter, or

$$F_N = \frac{G m_b m_J}{R_J^2}$$

Now $\mu = \frac{F_f}{F_N}$, so $F_{fJ} = \mu F_N = \mu \frac{G m_b m_J}{R_J^2}$

But $\mu = \frac{F_f}{m_b g}$ so $F_{fJ} = \frac{F_f G m_b m_J}{m_b g R_J^2} = \frac{(40.0\text{ N})(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{s}^2)(1.90 \times 10^{27}\text{ kg})}{(9.80\text{ m/s}^2)(7.18 \times 10^7\text{ m})^2}$

$= 100$ N

Note, the mass of the block divided out.

17. Use the information for Earth from Table 8-1 to calculate the mass of the sun using Newton's variations of Kepler's third law.

$T^2 = \left[\frac{4\pi^2}{Gm}\right] r^3$, so $mT^2 = \left[\frac{4\pi^2}{G}\right] r^3$ and

$$m = \left[\frac{4\pi^2}{G}\right]\frac{r^3}{T^2} = \left[\frac{4\pi^2}{6.670 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{s}^2}\right]\frac{(1.4957 \times 10^{11}\text{ m})^3}{(3.156 \times 10^7\text{ s})^2} = 1.989 \times 10^{30}\text{ kg}$$

18. Mimas, a moon of Saturn, has an orbital radius of 1.87 × 10^8 m and an orbital period of about 23 hours. Use Newton's variation of Kepler's third law and this data to find the mass of Saturn.

$$T^2 = \left[\frac{4\pi^2}{Gm}\right] r^3, \text{ so } m = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(1.87 \times 10^8\text{ m})^3}{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(82\,800\text{ s})^2} = 5.6 \times 10^{26}\text{ kg}$$

19. Use Newton's variation of Kepler's third law to find the mass of Earth. The moon is 3.9 × 10^8 m away from Earth and the moon has a period of 27.33 days. Compare this mass to the mass found in Problem 14.

$$T^2 = \left[\frac{4\pi^2}{Gm}\right] r^3 \text{ so } m = \left[\frac{4\pi^2}{G}\right]\frac{r^3}{T^2} = \left[\frac{4\pi^2}{6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2}\right]\frac{(3.9 \times 10^8\text{ m})^3}{(2.361 \times 10^6\text{ s})^2} = 6.3 \times 10^{24}\text{ kg}$$

very close

20. A geosynchronous satellite appears to remain over one spot on Earth. A geosynchronous satellite has an orbital radius of 4.23 × 10^7 m.

a. Calculate its speed in orbit.

$$v = \sqrt{\frac{Gm_e}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.979 \times 10^{24}\text{ kg})}{(4.23 \times 10^7\text{ m})}} = \sqrt{9.43 \times 10^6\text{ m}^2/\text{s}^2}$$

$= 3.07 \times 10^3$ m/s or 3.07 km/s

b. Calculate its period.

$$T = 2\pi\sqrt{\frac{r^3}{Gm_e}} = 2\pi\sqrt{\frac{(4.23 \times 10^7\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.979 \times 10^{24}\ \text{kg})}} = 2\pi\sqrt{1.90 \times 10^8\ \text{s}^2}$$
$$= 8.66 \times 10^4\ \text{s or } 24.0\ \text{h}$$

21. On July 19, 1969, Apollo II's orbit around the moon was adjusted to an average orbit of 111 km. The radius of the moon is 1785 km and the mass of the moon is 7.3×10^{22} kg.

$r = 111\ \text{km} + 1785\ \text{km} = 1896\ \text{km}.$

a. How many minutes did it take to orbit once?

$$T = 2\pi\sqrt{\frac{r^3}{Gm}} = 2\pi\sqrt{\frac{(1896 \times 10^3\ \text{m})^3}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.3 \times 10^{22}\ \text{kg})}} = 2\pi\sqrt{1.4 \times 10^6\ \text{s}^2}$$
$$= 7.4 \times 10^3\ \text{s} = 1.2 \times 10^2\ \text{min}$$

b. At what velocity did it orbit the moon?

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.3 \times 10^{22}\ \text{kg})}{1896 \times 10^3\ \text{m}}} = \sqrt{2.6 \times 10^6\ \text{m}^2/\text{s}^2} = 1.6 \times 10^3\ \text{m/s}$$

22. The asteroid Ceres has a mass 7×10^{20} kg and a radius of 500 km.

a. What is g on the surface?

$$g = \frac{Gm}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7 \times 10^{20}\ \text{kg})}{(500 \times 10^3\ \text{m})^2} = 0.2\ \text{m/s}^2$$

b. How much would an 85-kg astronaut weigh on Ceres?

$W = mg = (85\ \text{kg})(0.2\ \text{m/s}^2) = 17 = 2 \times 10^1\ \text{N}$

23. The radius of Earth is about 6.40×10^3 km. A 7.20×10^3-N spacecraft travels away from Earth. What is the weight of the spacecraft at the following distances from Earth's surface?

$R_E = 6.40 \times 10^3\ \text{km}$

$W \propto \frac{1}{d^2}$

a. 6.40×10^3 km

$d = R_E + R_E = 2R_E.$

Therefore, $W = \frac{1}{4}$ original weight $= \frac{1}{4}(7.20 \times 10^3\ \text{N}) = 1.80 \times 10^3\ \text{N}$

b. 1.28×10^4 km

$d = R_E + 2R_E = 3R_E;$

$W = \frac{1}{9}(7.20 \times 10^3\ \text{N}) = 800\ \text{N}$

Chapter Review Problems

24. How high does a rocket have to go above Earth's surface until its weight is half what it would be on Earth?

Now $W \propto \frac{1}{d^2}$ so $d \propto \sqrt{\frac{1}{W}}$

If the weight is $\frac{1}{2}$ the distance is $\sqrt{2}$ or $d = \sqrt{2}\ (6.40 \times 10^6\ \text{m}) = 9.05 \times 10^6\ \text{m}$

$9.05 \times 10^6\ \text{m} - 6.40 \times 10^6\ \text{m} = 2.65 \times 10^6\ \text{m} = 2.65 \times 10^3\ \text{km}.$

25. The formula for the period of a pendulum, T, is $T = 2\pi\sqrt{l/g}$.

$$g = \frac{Gm}{R^2} = \frac{(6.670 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.34 \times 10^{22}\ \text{kg})}{(1.74 \times 10^6\ \text{m})^2} = 1.62\ \text{m/s}^2$$

a. What would be the period of a 2.0 m long pendulum on the moon's surface? The moon's mass is 7.34×10^{22} kg and its radius is 1.74×10^6 m.

$$T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{(2.0\ \text{m})}{(1.62\ \text{m/s}^2)}} = 7.0\ \text{s}$$

b. What is the period of this pendulum on Earth?

$$T = 2\pi\sqrt{l/g} = 2\pi\sqrt{\frac{(2.0\ \text{m})}{(9.80\ \text{m/s}^2)}} = 2.8\ \text{s}$$

26. A 1.25–kg book in space has a weight of 8.35 N. What is the value of the gravitational field at that location?

$g = F/m = (8.35\ \text{N})/(1.25\ \text{kg}) = 6.68\ \text{N/kg}$

27. The moon's mass is 7.34×10^{22} kg and it is 3.8×10^8 m away from Earth. Earth's mass can be found in Table 8–1.

a. Calculate the gravitational force of attraction between the two.

$$F = \frac{Gm_e m_m}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.979 \times 10^{24}\ \text{kg})(7.34 \times 10^{22}\ \text{kg})}{(3.8 \times 10^8\ \text{m})^2} = 2.0 \times 10^{20}\ \text{N}$$

b. Find the Earth's gravitational field at the moon.

$$g = F/m = \frac{2.0 \times 10^{20}\ \text{N}}{7.34 \times 10^{22}\ \text{kg}} = 0.0028\ \text{N/kg}$$

28. Earth's gravitational field is 7.83 N/kg at the altitude of the space shuttle. What is the size of the force of attraction between a student, mass of 45.0 kg, and Earth?

$g = F/m$, so $F = mg = (45.0)(7.83) = 352\ \text{N}$

Supplemental Problems (Appendix B)

1. Comet Halley returns every 74 years. Find the average distance of the comet from the sun.

   $(r_a/r_b)^3 = (T_a/T_b)^2$, so
   $r_a^3 = r_b^3(T_a/T_b)^2 = (1.0\ \text{AU})^3(74\ \text{y}/1.0\ \text{y})^2 = 5.48 \times 10^3\ \text{AU}^3$,
   so $r_a = 18\ \text{AU}$ or $18(1.5 \times 10^{11}\ \text{m}) = 2.7 \times 10^{12}\ \text{m}$

2. Area is measured in m^2, so the rate at which area is swept out by a planet or satellite is measured in m^2/s.

   The total area for one orbit is πr^2 and the total time is one period T. The rate is $\pi r^2/T$.

   a. How fast is area swept out by Earth in its orbit about the sun. See Table 8–1.

   $r = 1.49 \times 10^{11}$ m and $T = 3.156 \times 10^7$ s, so
   $\pi r^2/T = \pi(1.49 \times 10^{11}\ \text{m})^2/\text{s}/(3.156 \times 10^7\ \text{s}) = 2.21 \times 10^{15}\ \text{m}^2/\text{s}$

   b. How fast is area swept out by the moon in its orbit about Earth? Use 3.9×10^8 m as the average distance between the Earth and the moon, and 27.3 days as the moon's period.

   $\pi(3.9 \times 10^8\ \text{m})^2/(2.36 \times 10^7\ \text{s}) = 2.0 \times 10^{11}\ \text{m}^2/\text{s}$

3. You wish to launch a satellite that will remain above the same spot on Earth's surface. This means the satellite must have a period of exactly one day. Calculate the radius of the circular orbit this satellite must have. Hint: The moon also circles Earth and both the moon and satellite will obey Kepler's third law. The moon 3.8 × 10⁸ m from Earth and its period is 27.33 days.

   $$\left[\frac{T_s}{T_m}\right]^2 = \left[\frac{r_s}{r_m}\right]^3,\ \text{so}\ r_s^3 = \left[\frac{T_s}{T_m}\right]^2 r_m^3 = \left[\frac{1000\ \text{dy}}{27.33\ \text{dy}}\right]^2 (3.8 \times 10^8\ \text{m})^3 = 7.35 \times 10^{22}\ \text{m}^3$$

   so $r_s = 4.2 \times 10^7$ m

4. The mass of an electron is 9.1×10^{-31} kg. The mass of a proton is 1.7×10^{-27} kg. They are about 1.0×10^{-10} m apart in a hydrogen atom. What gravitational force exists between the proton and the electron of a hydrogen atom?

   $$F = \frac{Gm_e m_p}{d^2} = \frac{(6.67 \times 10^{-4}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.1 \times 10^{-31}\ \text{kg})(1.7 \times 10^{-27}\ \text{kg})}{(1.0 \times 10^{-10}\ \text{m})^2} = 1.0 \times 10^{-47}\ \text{N}$$

5. Two 1.00–kg masses have their centers 1.00 m apart. What is the force of attraction between them?

   $$F_g = G\frac{m_1 m_2}{d^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.00\ \text{kg})(1.00\ \text{kg})}{(1.00\ \text{m})^2} = 6.67 \times 10^{-11}\ \text{N}$$

6. Two satellites of equal mass are put into orbit 30 m apart. The gravitational force between them is 2.0×10^{-7} N.

   a. What is the mass of each sattelite?

   $$F = G\frac{m_1 m_2}{r_2}$$

   $$m = \sqrt{\frac{Fr^2}{G}} = \sqrt{\frac{(2.0 \times 10^{-7}\ \text{N})(30\ \text{m})^2}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}} = \sqrt{2.698 \times 10^6\ \text{kg}^2} = 1.6 \times 10^3\ \text{kg}$$

b. What is the initial acceleration given to each satellite by the gravitational force?

$F = ma$

$$a = \frac{F}{m} = \frac{2.0 \times 10^{-7}\ \text{N}}{1.6 \times 10^{3}\ \text{kg}} = 1.3 \times 10^{-10}\ \text{m/s}^2$$

7. Two large spheres are suspended close to each other. Their centers are 4.0 m apart. One sphere weighs 9.8×10^2 N. The other sphere has a weight of 1.96×10^2 N. What is the gravitational force between them?

$$m_1 = \frac{W}{g} = \frac{9.8 \times 10^2\ \text{N}}{9.8\ \text{m/s}^2} = 1.0 \times 10^2\ \text{kg}$$

$$m_2 = \frac{W}{g} = \frac{1.96 \times 10^2\ \text{N}}{9.8\ \text{m/s}^2} = 2.0 \times 10^1\ \text{kg}$$

$$F = G\frac{m_1 m_2}{d^2} = [(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.0 \times 10^2\ \text{kg})(2.0 \times 10^1\ \text{kg})] \div (4.0\ \text{m})^2$$

$$= 8.3 \times 10^{-9}\ \text{N}$$

8. If the centers of Earth and the Moon are 3.9×10^8 m apart, the gravitational force between them is about 1.9×10^{20} N. What is the approximate mass of the moon?

$$F = G\frac{m_1 m_2}{r^2}$$

$$m_m = \frac{F r^2}{G m_e} = \frac{(1.9 \times 10^{20}\ \text{N})(3.9 \times 10^8\ \text{m})^2}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.0 \times 10^{24}\ \text{kg})} = 7.2 \times 10^{22}\ \text{kg}$$

9. **a.** What is the gravitational force between two 8.00-kg spherical masses that are 5.0 m apart?

$$F = G\frac{m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.0\ \text{kg})(8.0\ \text{kg})}{(5.0\ \text{m})^2} = 1.7 \times 10^{-10}\ \text{N}$$

b. What is the gravitational force between them when they are 5.0×10^1 m apart?

$$F = G\frac{m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.0\ \text{kg})(8.0\ \text{kg})}{(5.0 \times 10^1\ \text{m})^2} = 1.7 \times 10^{-12}\ \text{N}$$

10. A satellite is placed in a circular orbit of 1.0×10^7 m radius with a period of 9.9×10^3 s. Calculate the mass of Earth. **Hint:** Gravity supplies the needed centripetal force for such a satellite. Scientists have actually measured the mass of Earth this way.

$F = m_s v^2/r = G m_s m_e/r^2$. Since, $v = 2\pi r/T$, $\left[\frac{m_s}{r}\right]\left[\frac{4\pi^2 r^2}{T^2}\right] = \frac{G m_s m_e}{r^2}$

$m_e = 4\pi^2 r^3/GT^2 = (4)(3.14)^2(1.0 \times 10^7\ \text{m})^3/(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(9.9 \times 10^3\ \text{s})^2$
$m_e = 6.0 \times 10^{24}$ kg

11. If you weigh 637 N on Earth's surface, how much would you weigh on the planet Mars? (Mars has a mass of 6.37×10^{23} kg and a radius of 3.43×10^6 m.)

$m = W/g = (637\ \text{N})/(9.80\ \text{m/s}^2) = 65.0$ kg
$F = G m_1 m_2/d^2 = (6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(65.0\ \text{kg})(6.37 \times 10^{23}\ \text{kg})/(3.43 \times 10^6\ \text{m}) = 235$ N

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12. Using Newton's variation of Kepler's third law and information from Table 8–1, calculate the period of Earth's moon if the radius of orbit was twice the actual value of 3.9×10^8 m.

$$(T_p^2) = \left[\frac{4\pi^2}{GME}\right](R^3) = \left[\frac{4\pi^2}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.979 \times 10^{24}\ \text{kg})}\right](7.8 \times 10^8\ \text{m})^3$$

$T_p = 6.85 \times 10^6$ s or 79 days

13. Use the data from Table 8–1 to find the speed and period of a satellite that would orbit Mars 175 km above its surface.

$r = Rm + 175\ \text{km} = 3.56 \times 10^6$ m

$$v = \sqrt{\frac{GM_m}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})}{(3.56 \times 10^6\ \text{m})}}$$

$v = 3.47 \times 10^3$ m/s

$$T = 2\pi\sqrt{\frac{r^3}{GM_m}} = 2\pi\sqrt{\frac{(3.56 \times 10^6\ \text{m})^3}{(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(6.42 \times 10^{23}\ \text{kg})}}$$

$T = 6.45 \times 10^3$ s or 1.79 h

14. What would be the value of g, accleration of gravity, if Earth's mass was double its actual value, but its radius remained the same? If the radius was doubled, but the mass remained the same? If both the mass and radius were doubled?

$$g = \frac{GM_e}{R_e^2}$$

$2M_e \Rightarrow g = 19.6\ \text{m/s}^2$
$2R_e \Rightarrow g = 2.45\ \text{m/s}^2$
$2M_e$ and $2R_e \Rightarrow g = 4.9\ \text{m/s}^2$

15. What would be the strength of Earth's gravitational field at a point where an 80.0-kg astronaut would experience a 25% reduction in weight?

$W = mg = (80.0\ \text{kg})(9.80\ \text{m/s}^2) = 784$ N
$W_{reduced} = (784\ \text{N})(.75) = 588$ N

$$g_{reduced} = \frac{W_{reduced}}{m} = 588\ \text{N}/80.0\ \text{kg} = 7.35\ \text{m/s}^2$$

16. On the surface of the moon, a 91.0-kg physics teacher weighs only 145.6 N. What is the value of the moon's gravitational field at its surface?

$$W = mg,\ g = \frac{W}{m} = \frac{145.6\ \text{N}}{91.0\ \text{kg}} = 1.60\ \text{m/s}^2$$